Introduction to the Finite Element Method (FEM) – I

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Univerza v Ljubljani

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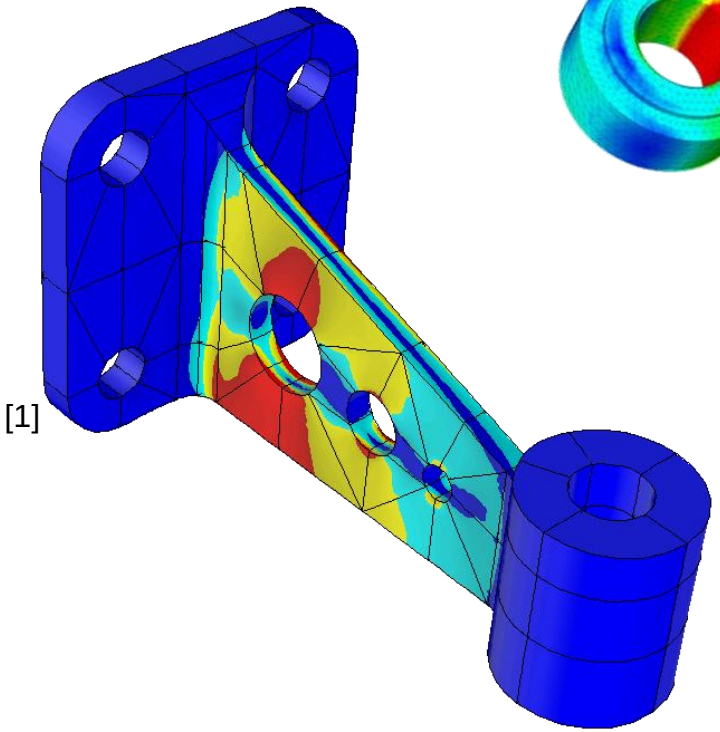
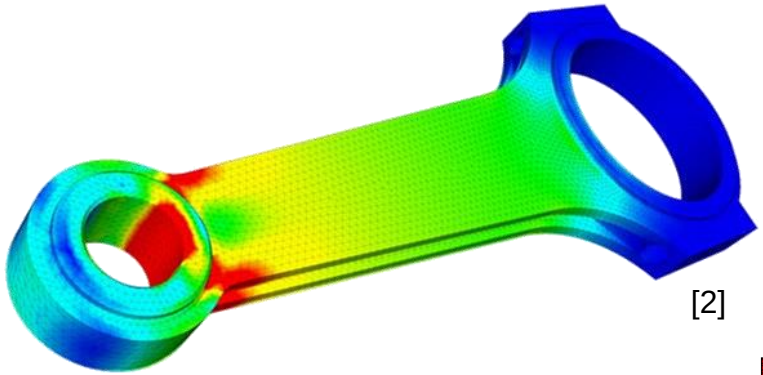
What is FEM?

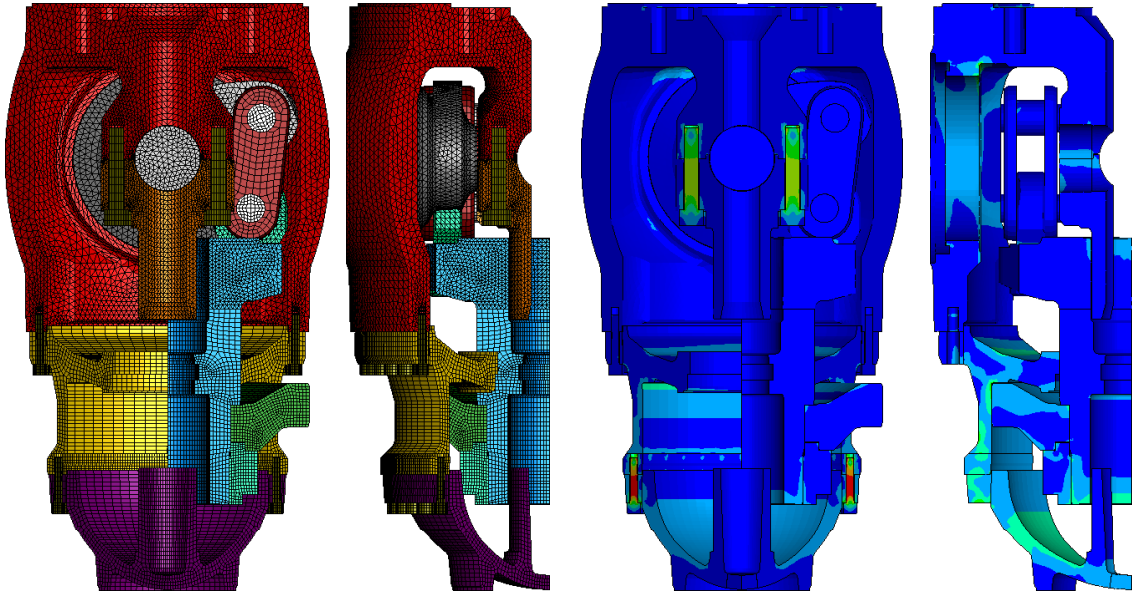
SCtrain | SUPERCOMPUTING KNOWLEDGE PARTNERSHIP



[1]



[2]



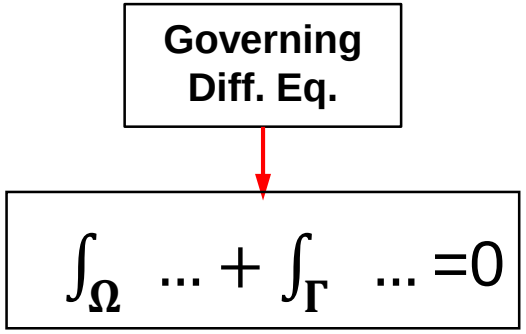
[1] https://manilsuri.umbc.edu/what-are-finite-elements/
[2] https://www.simscale.com/blog/2016/10/what-is-finite-element-method/

What is FEM?

Finite Element Method:

- is a procedure for obtaining numerical approximation to the solution of a boundary value problem.

What is FEM?



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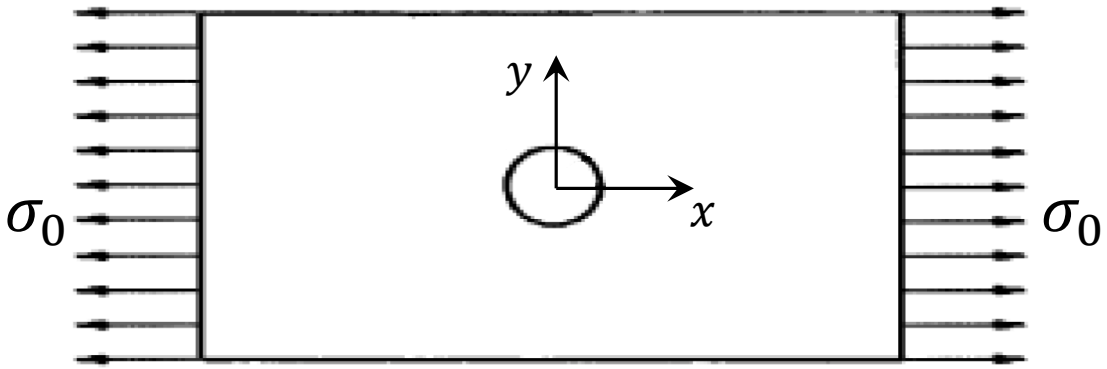
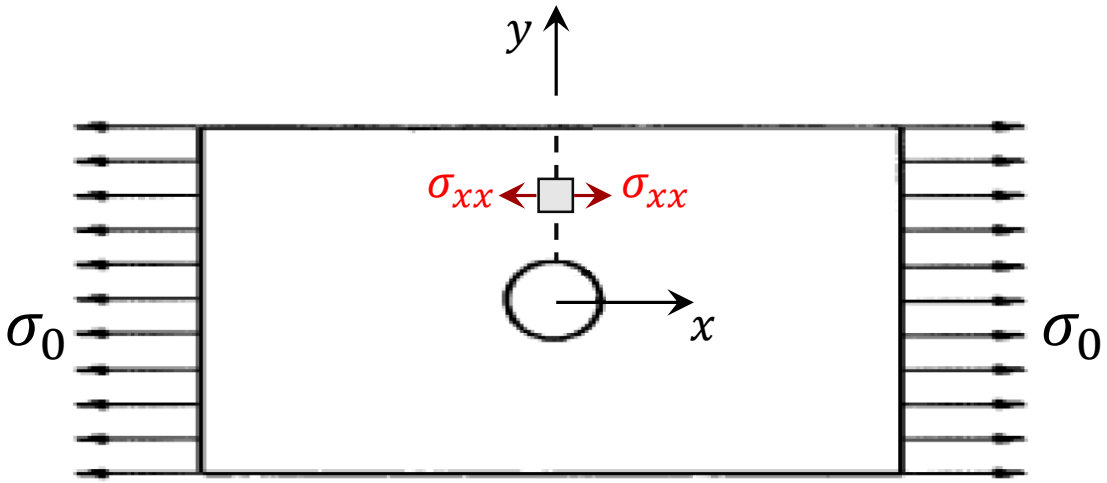


Governing Diff. Eq.

$$\int_{\Omega} \ldots + \int_{\Gamma} \ldots = 0$$

What is FEM?



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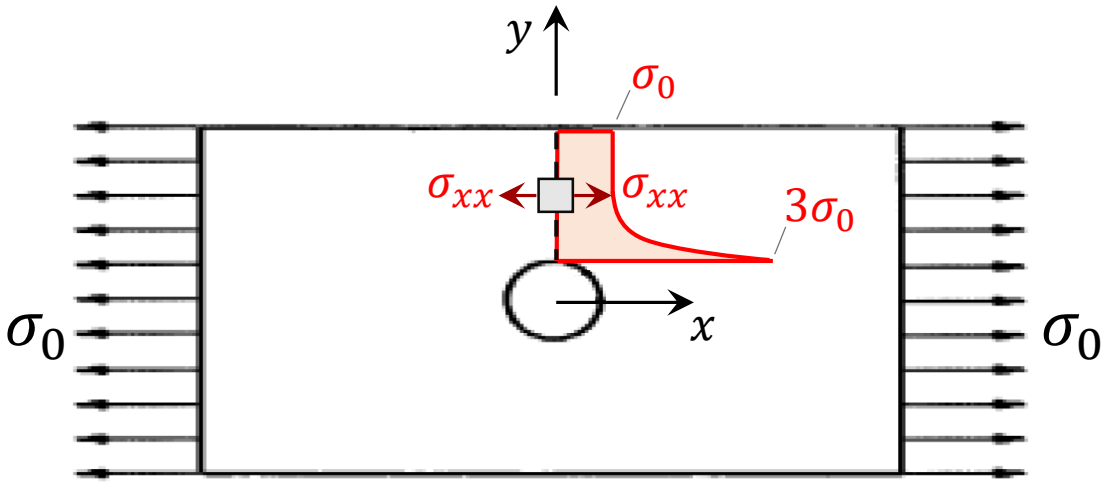

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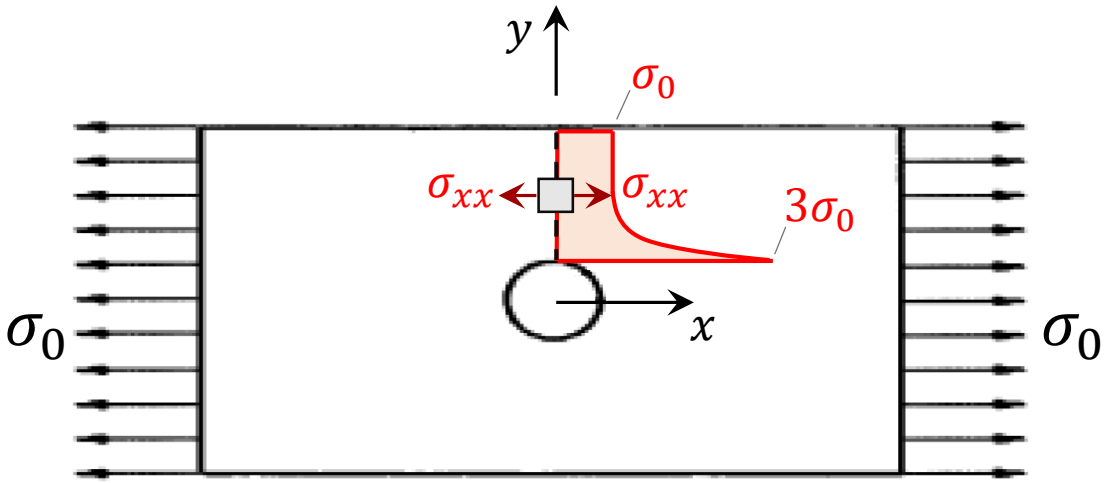

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$\sigma_{ij} = \sigma_{ij}(\boldsymbol{u})$

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… searching for ***u****(x,y)*

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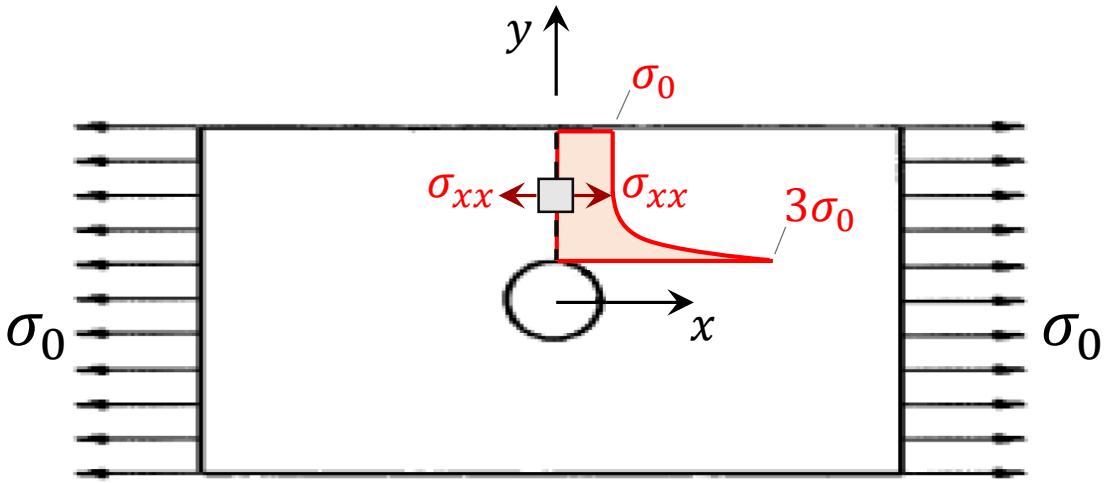

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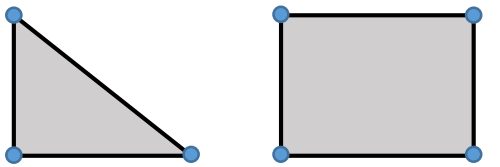
$c_0 + c_1 x + c_2 y$ $\quad$ $c_0 + c_1 x + c_2 y + c_3\, x\, y$



$\sigma_{ij} \propto \nabla \boldsymbol{u}$

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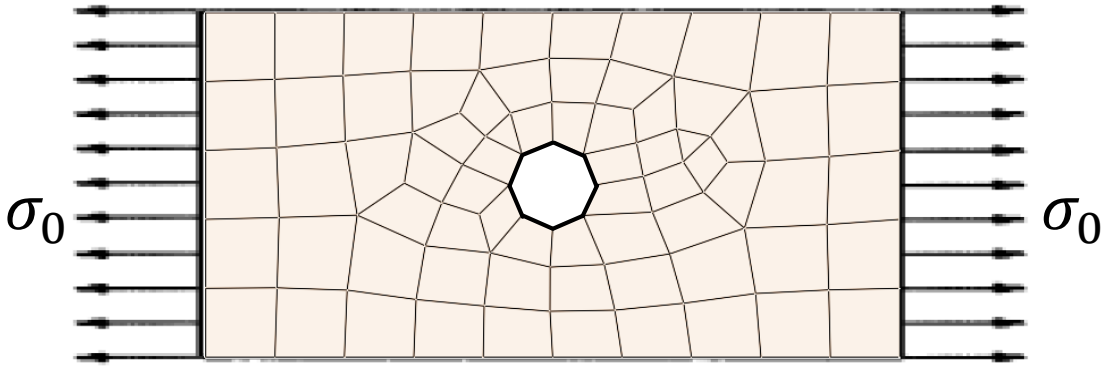

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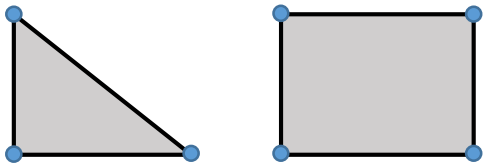

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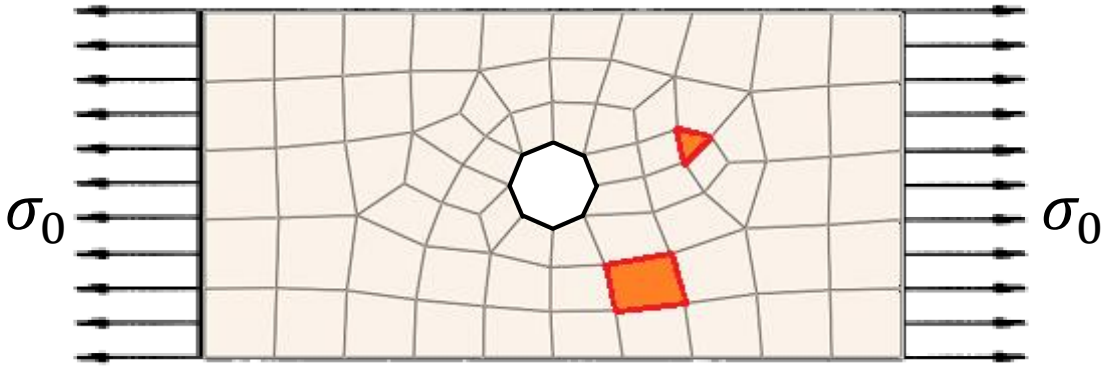

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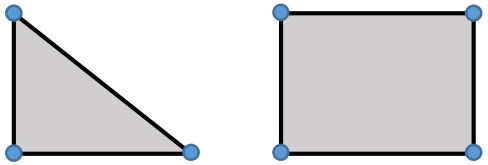

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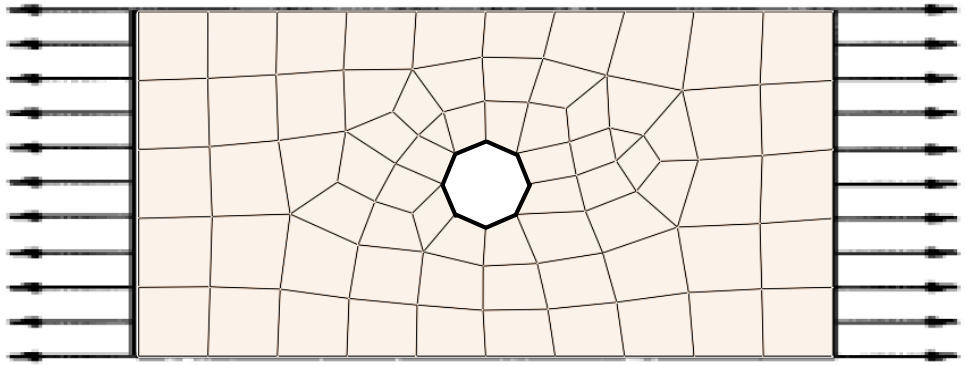

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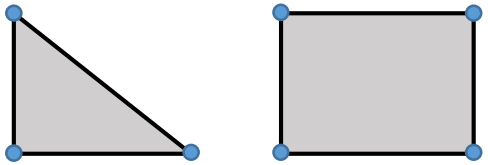

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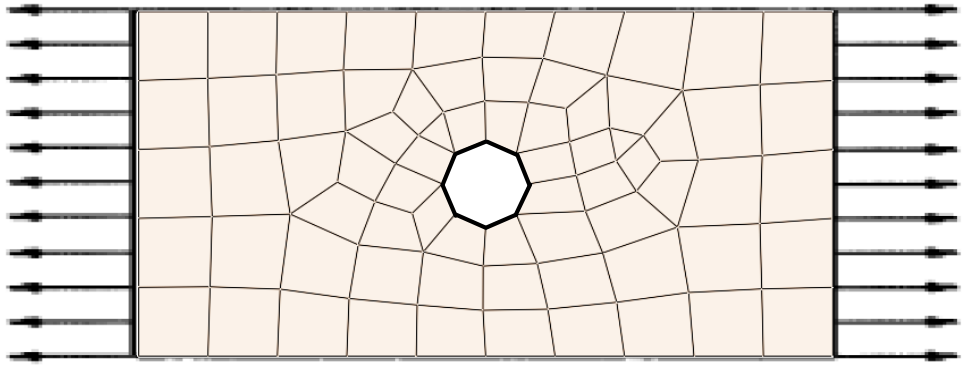

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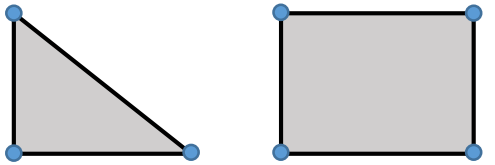

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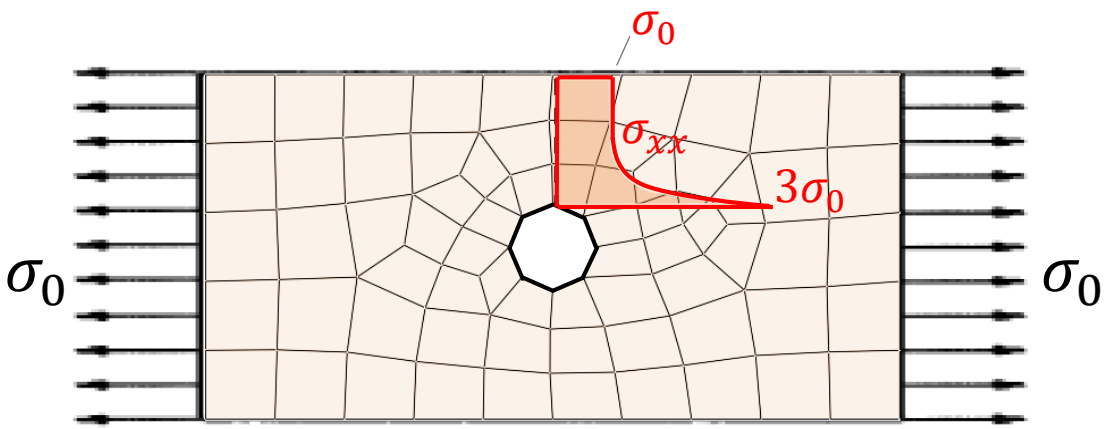

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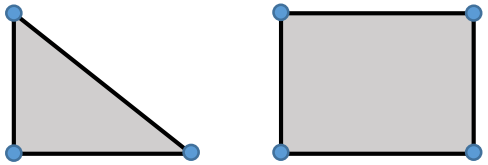

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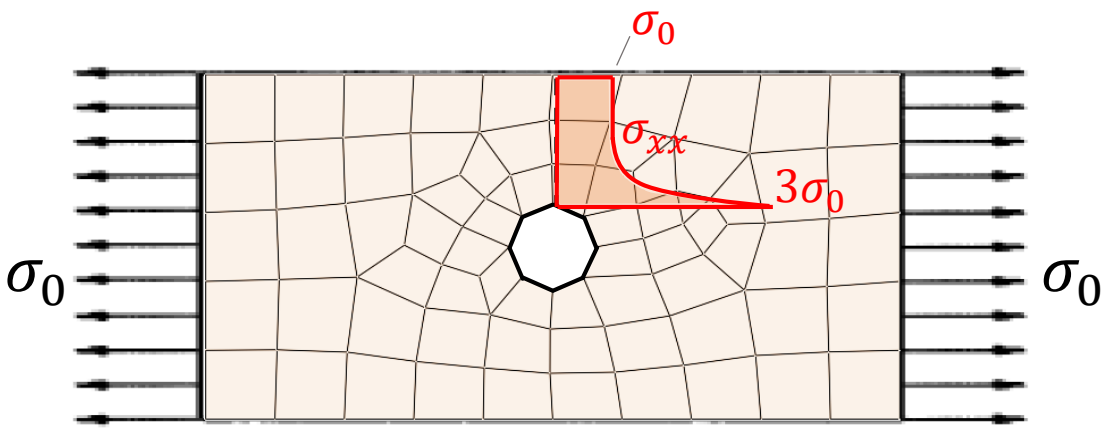

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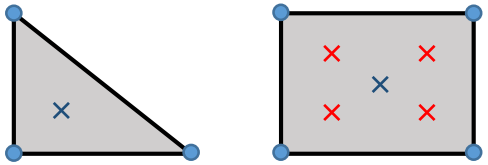

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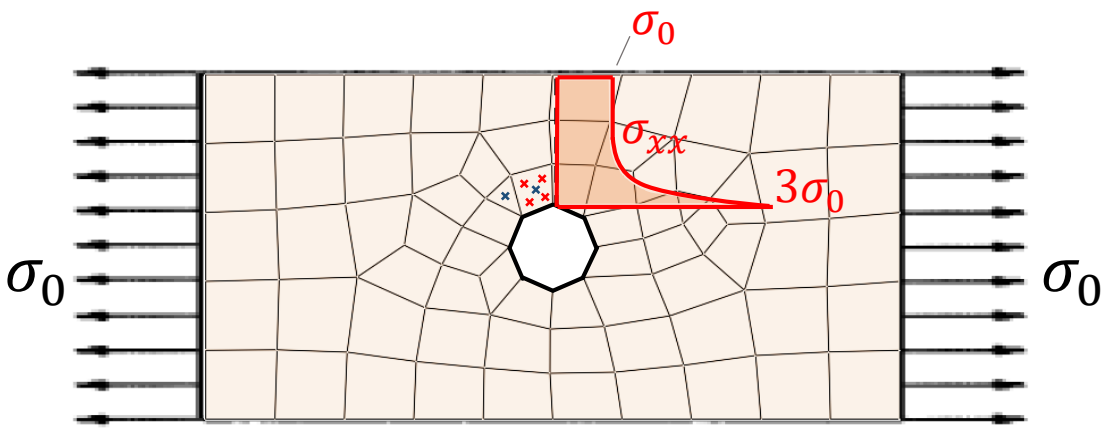

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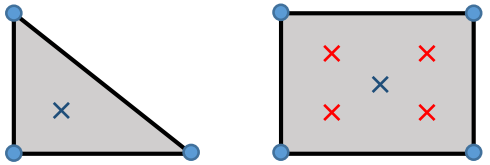

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GIGO

[1]

[1] https://www.r-bloggers.com/2019/08/new-course-learn-advanced-data-cleaning-in-r

Common FEM applications

- Mechanical/Aerospace/Civil/Automotive Engineering
- Structural/Stress Analysis
  - Static/Dynamic
  - Linear/Nonlinear
- Fluid Flow
- Heat Transfer
- Electromagnetic Fields
- Soil Mechanics
- Biomechanics

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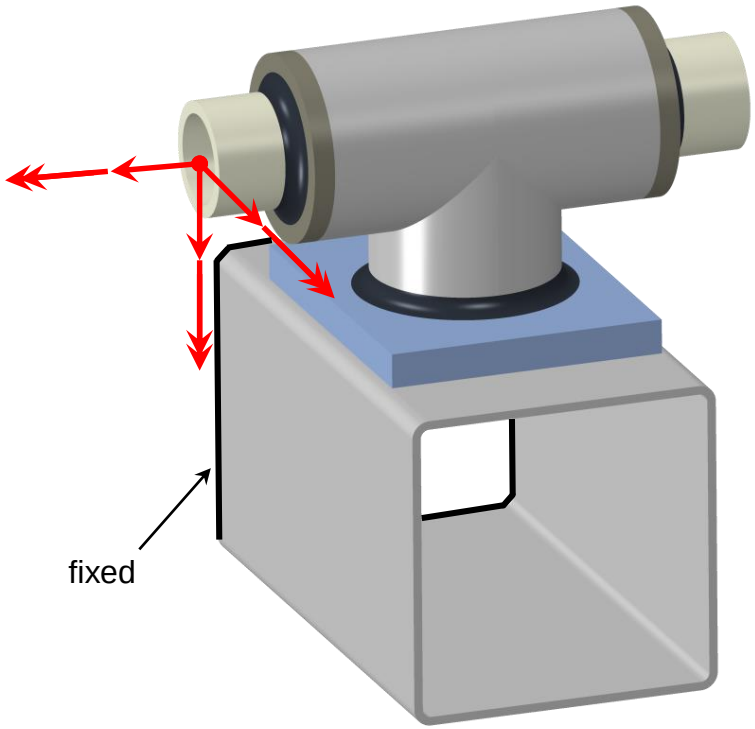
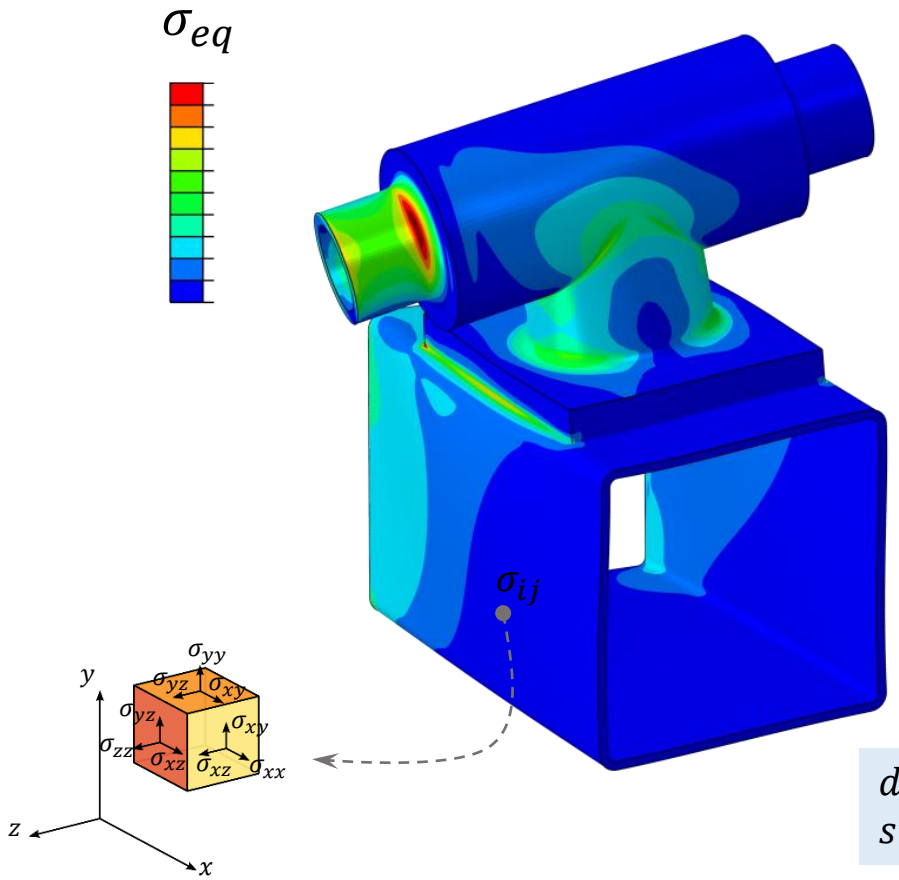
Common FEM applications

STATIC Stress analysis

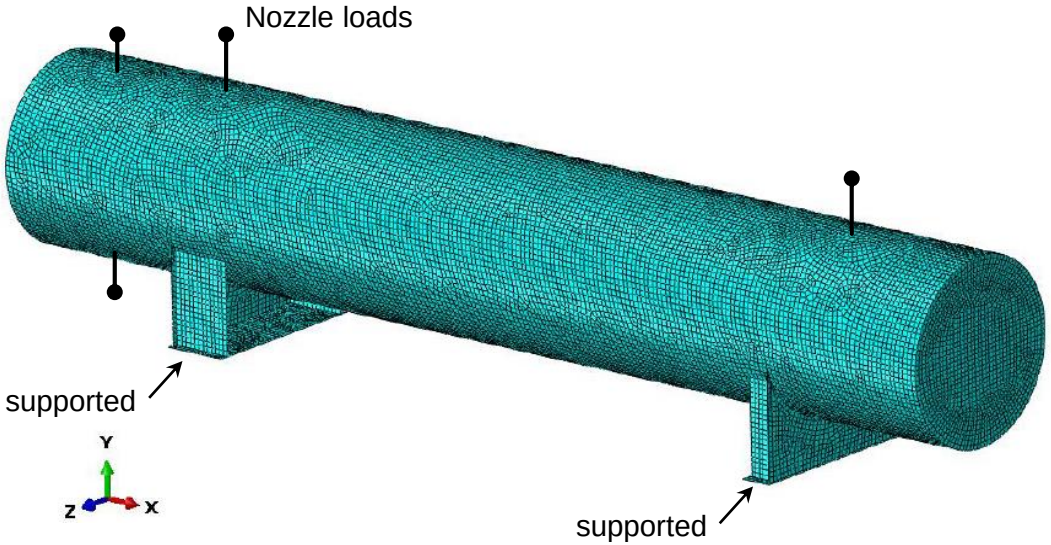


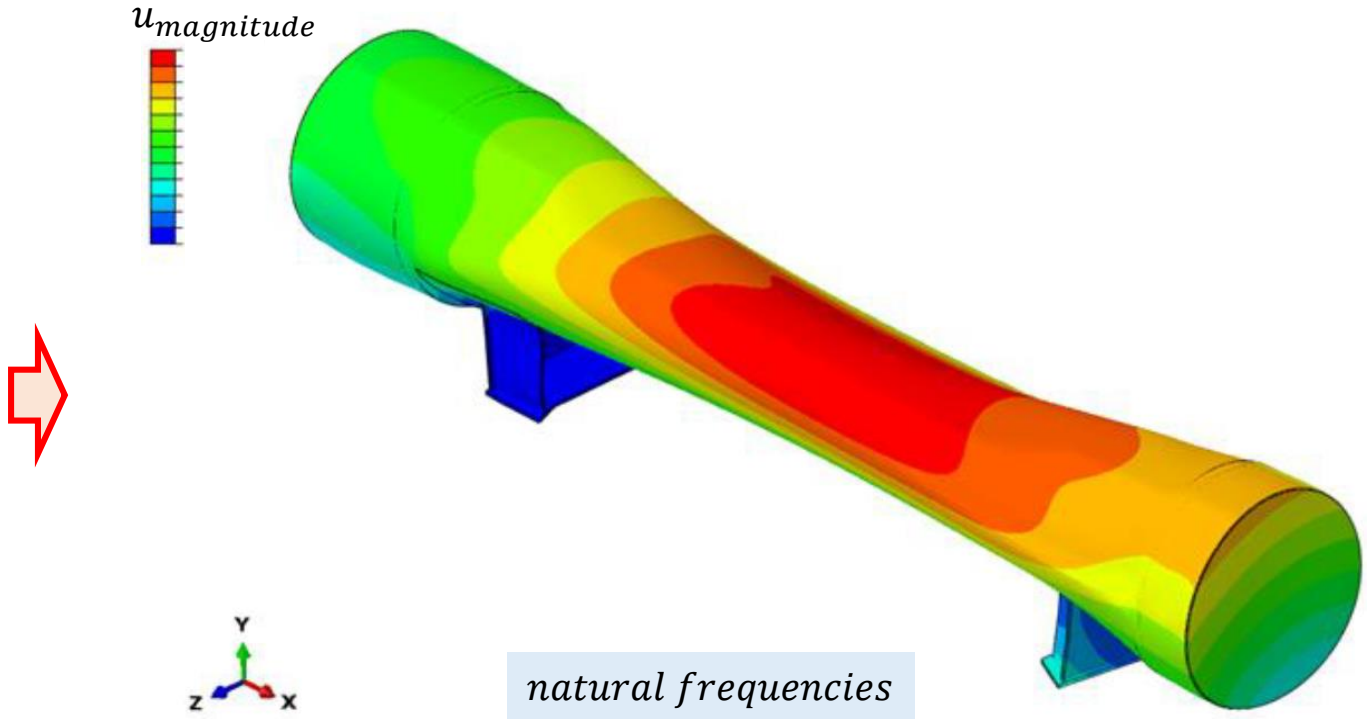


displacements
stresses

Common FEM applications

Modal analysis

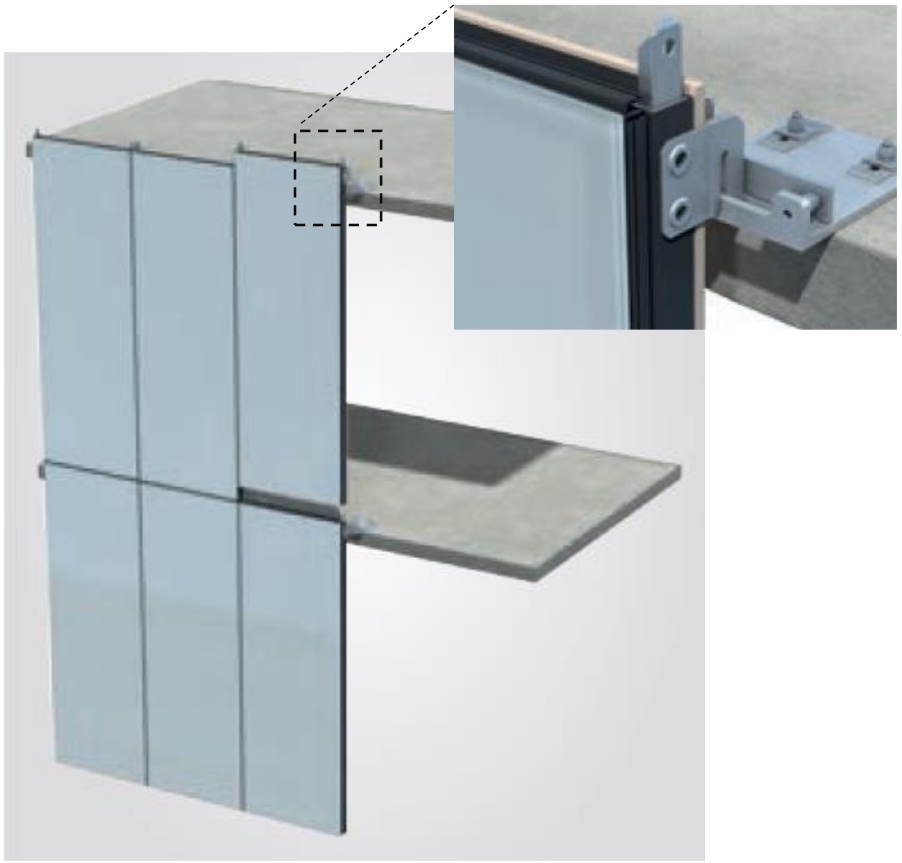


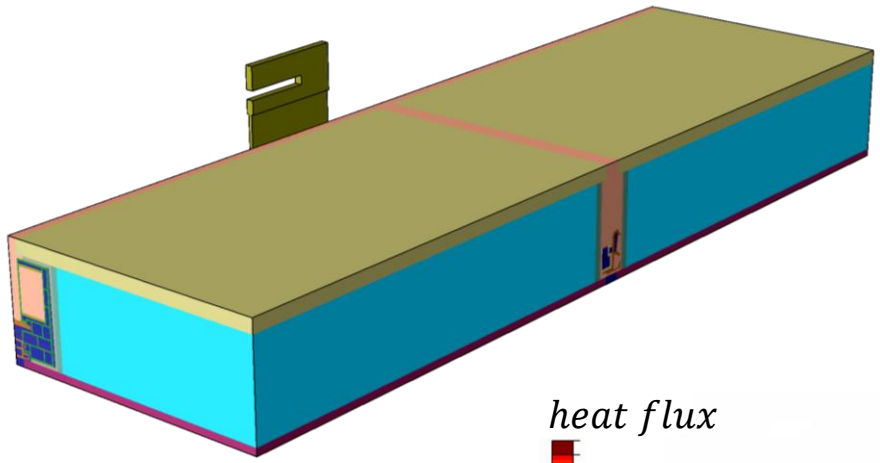


natural frequencies
mode shapes

Common FEM applications

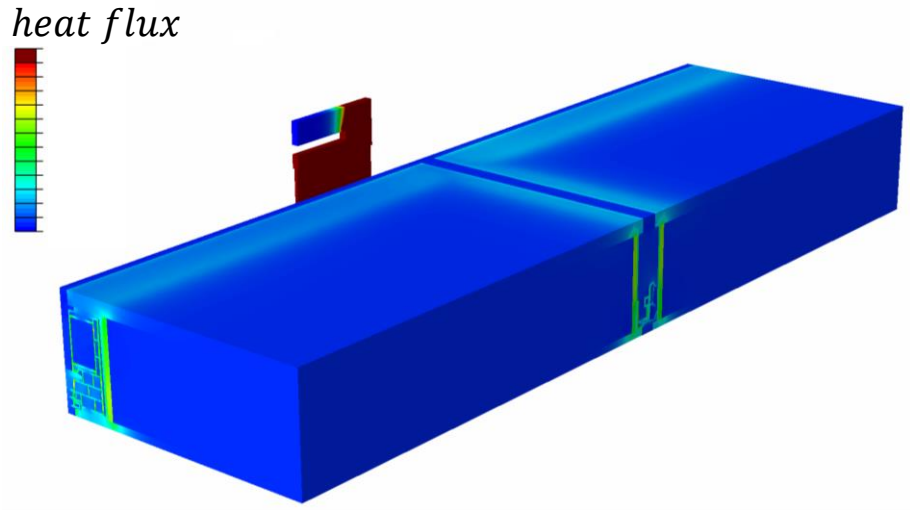
Heat transfer





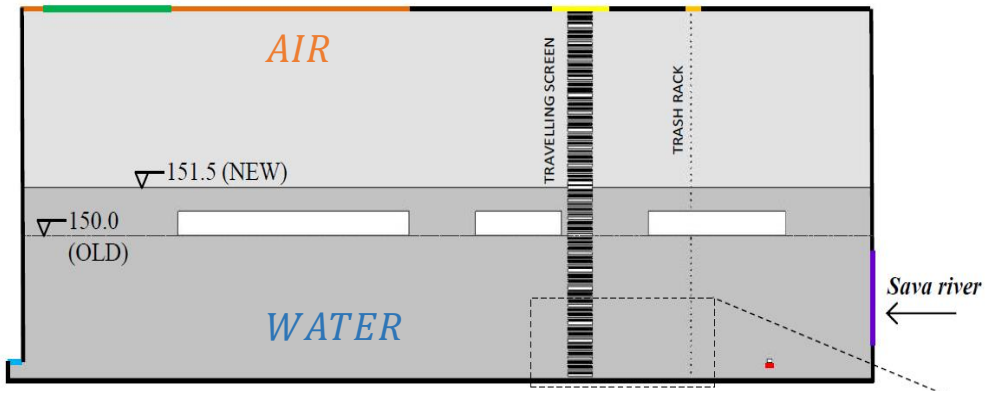
temperature
heat flux

heat flux



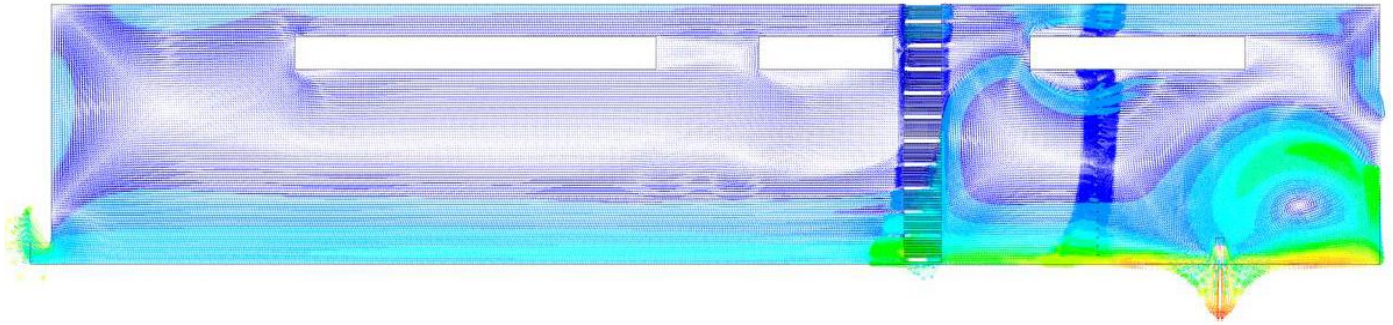
Common FEM applications

Transient thermo-hydraulic simulation (Fluid dynamics)

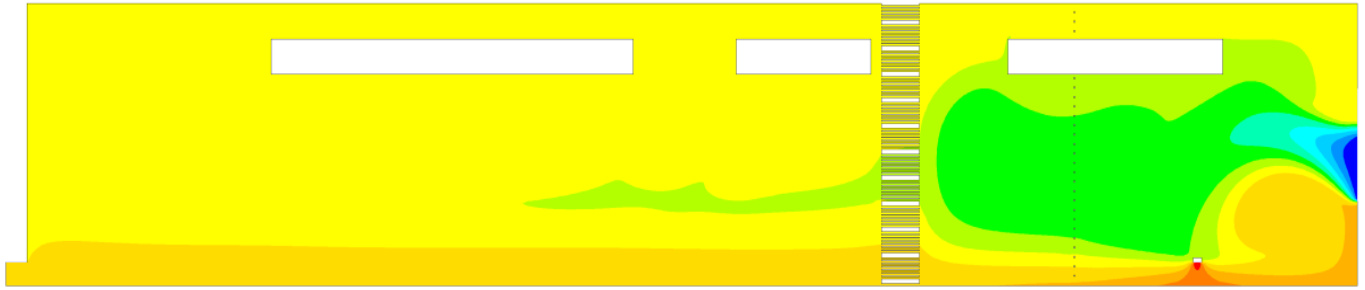


WATER domain

velocity



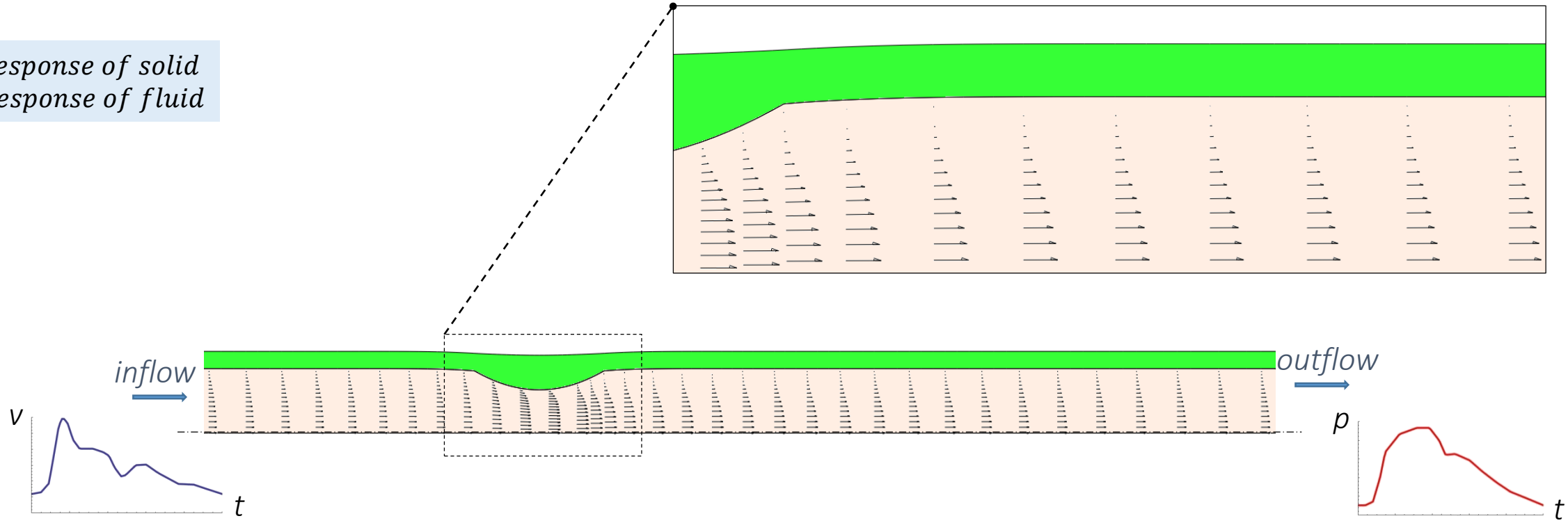
Temperature

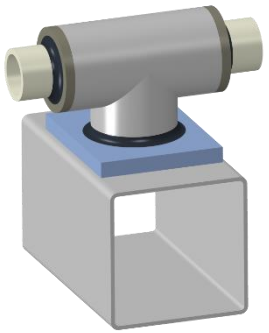


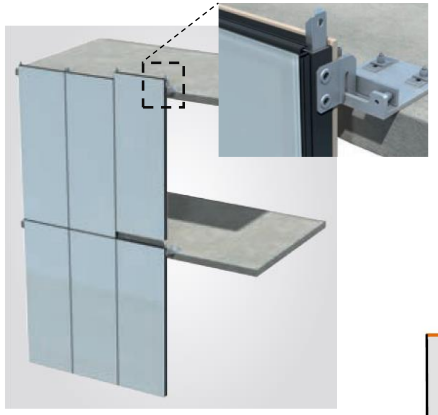
Common FEM applications

Coupled problems: Fluid-Structure Interaction (FSI)

$response\ of\ solid$
$response\ of\ fluid$







simulation

FEM simulation steps

1. Geometry
   - geometrical simplifications
   - reduction of dimensions
2. Physical properties
   - material properties
   - structural properties
3. Geometrical discretization
   - element type
   - meshing
4. Type of analysis
5. Loading and Boundary/Initial conditions
6. Presentation and analysis of results

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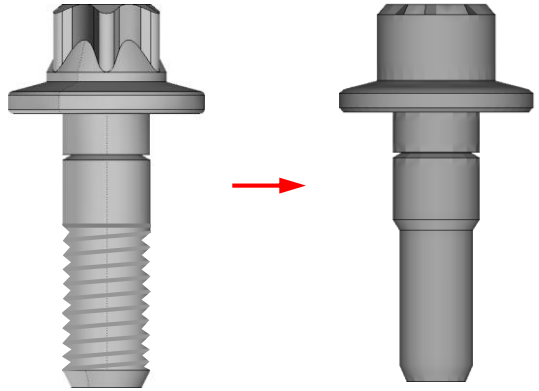

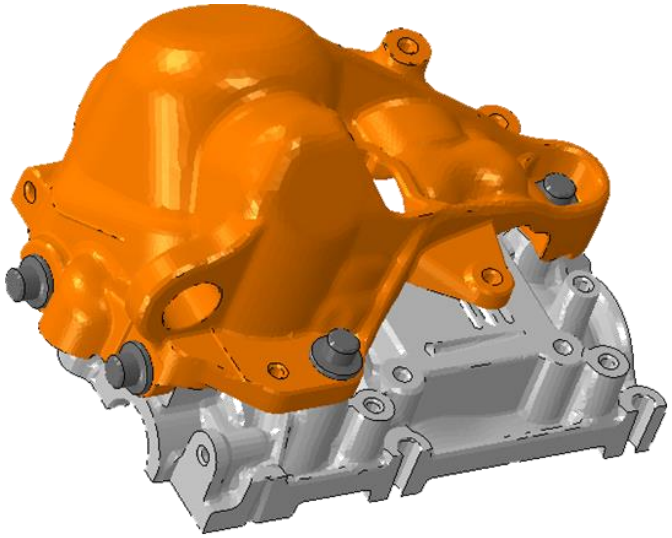

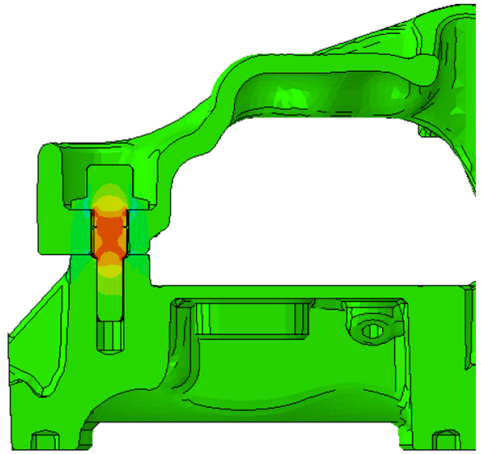

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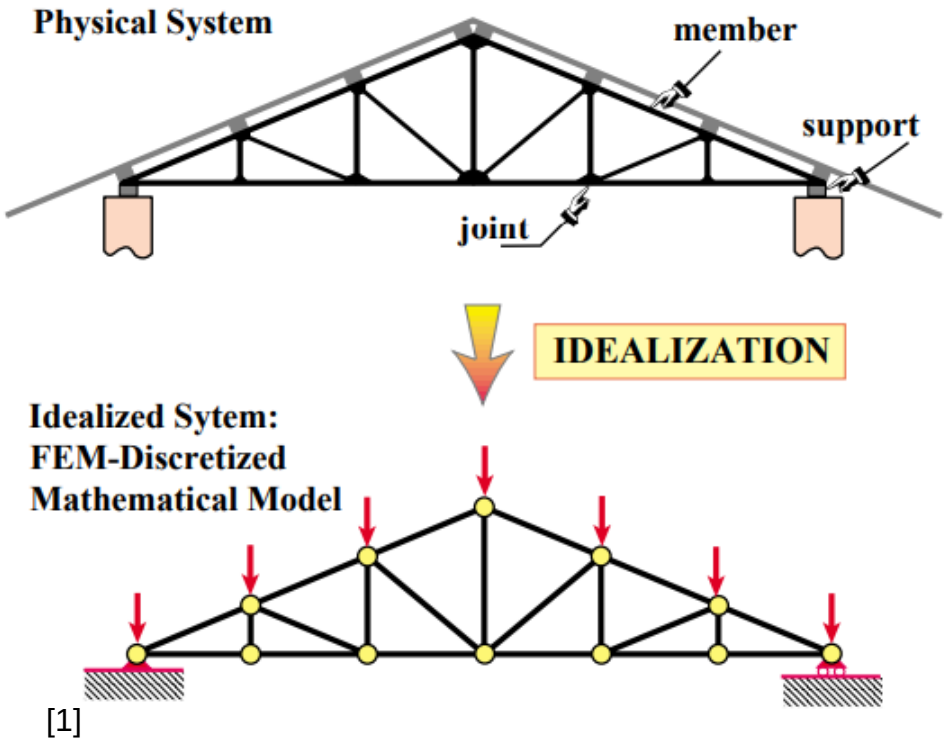

[1]

[1] Carlos A. Felippa, 2004, Introduction to Finite Element Methods. Available at: https://vulcanhammernet.files.wordpress.com/2017/01/ifem.pdf (06/2021)

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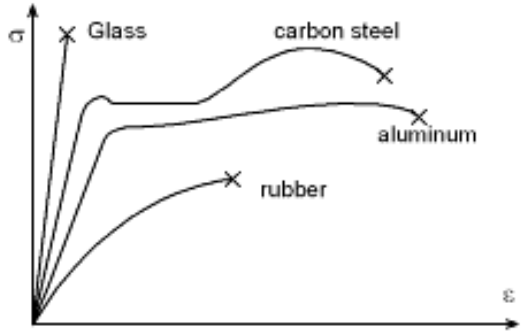

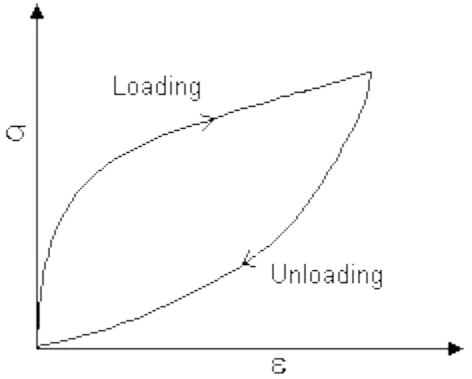

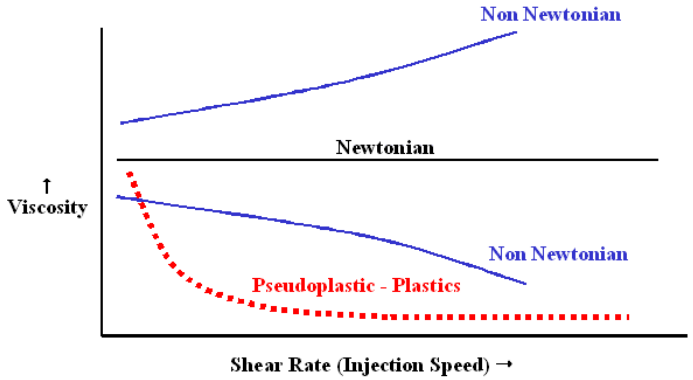

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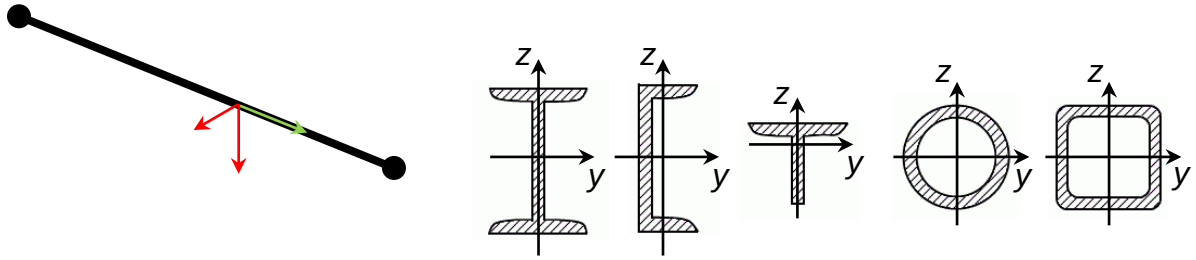
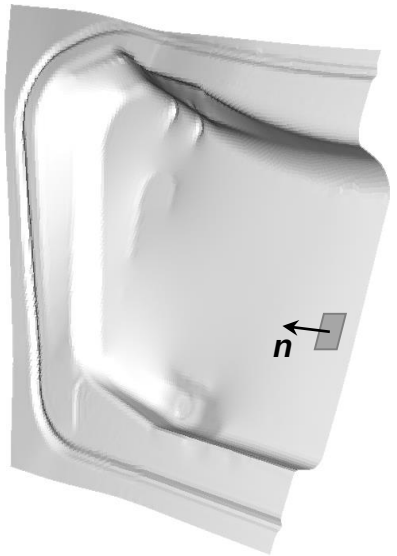
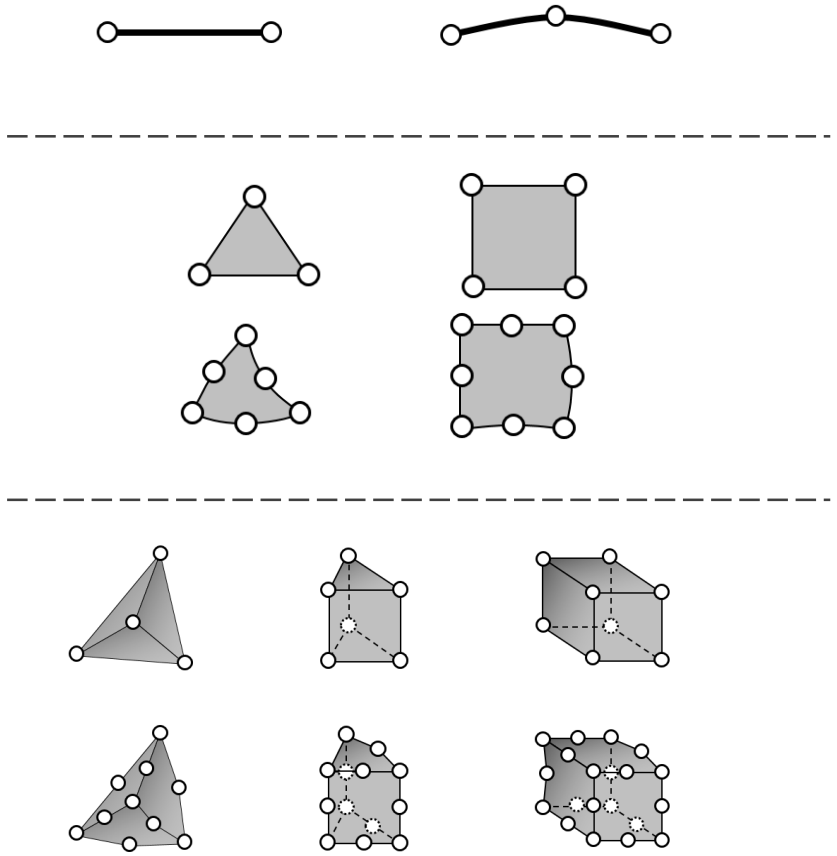
3. Geometrical discretization
   - element type
   - meshing
4. Type of analysis
5. Loading and Boundary/Initial conditions
6. Presentation and analysis of results





FEM simulation steps

1. Geometry
   - geometrical simplifications
   - reduction of dimensions
2. Physical properties
   - material properties
   - structural properties
3. **Geometrical discretization**
   - element type
   - meshing
4. Type of analysis
5. Loading and Boundary/Initial conditions
6. Presentation and analysis of results



FEM simulation steps

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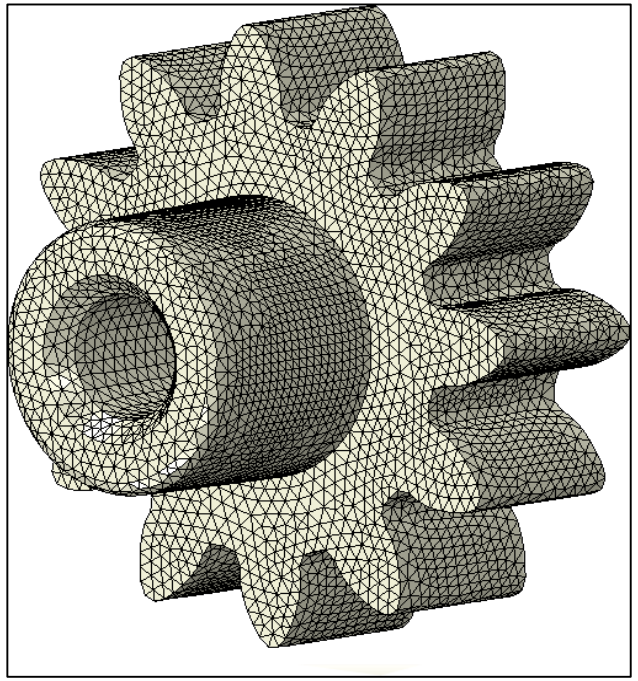

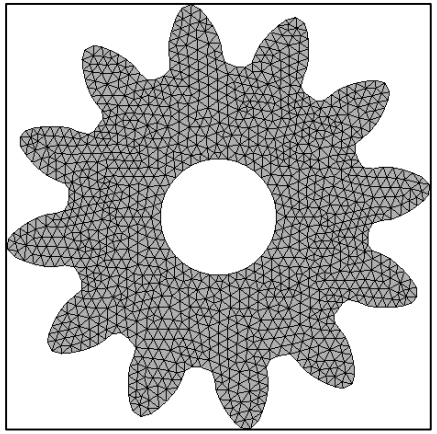

FEM simulation steps

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   - material properties
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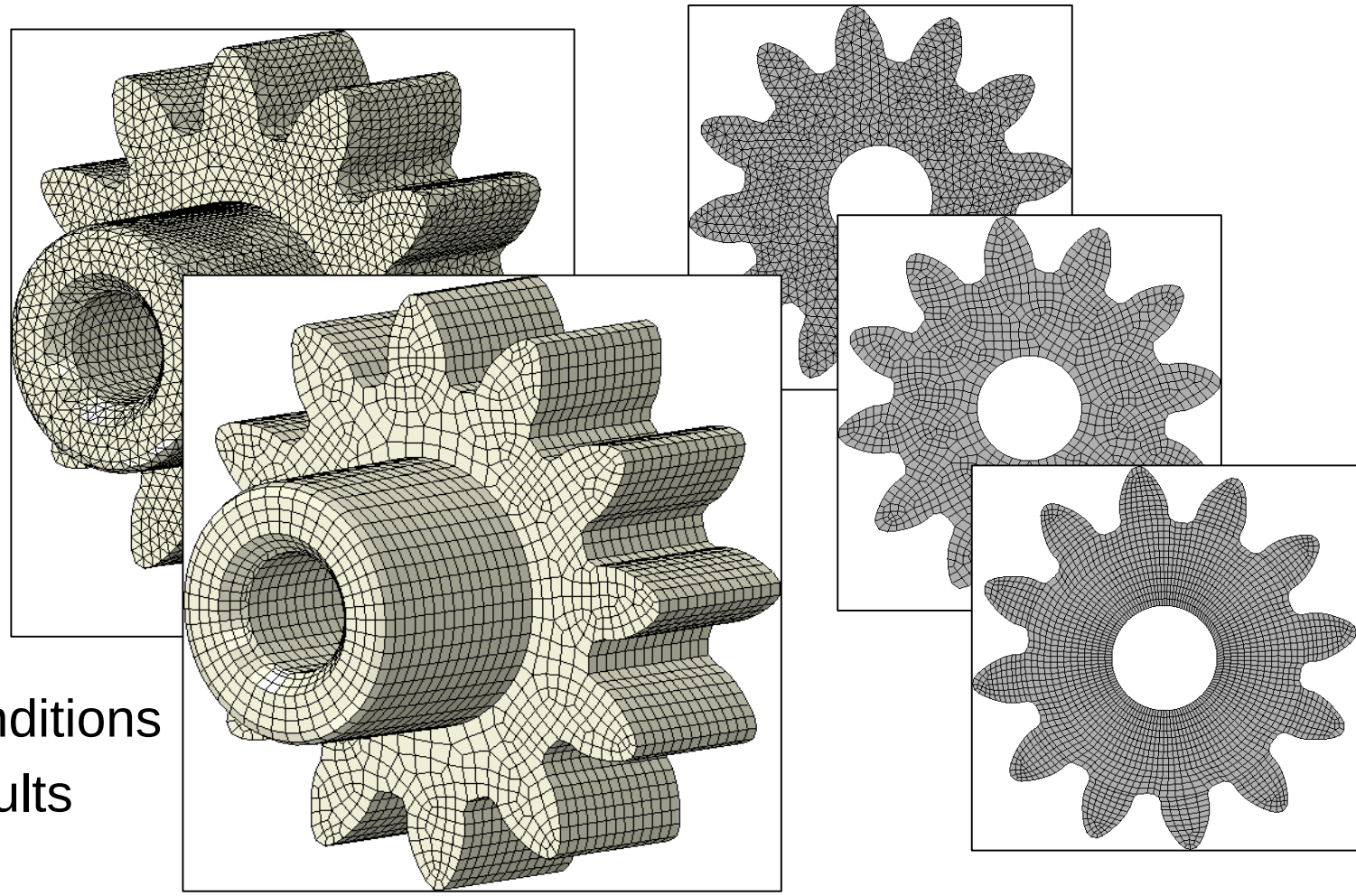

FEM simulation steps

1. Geometry
   - geometrical simplifications
   - reduction of dimensions
2. Physical properties
   - material properties
   - structural properties
3. Geometrical discretization
   - element type
   - meshing
4. **Type of analysis**
5. Loading and Boundary/Initial conditions
6. Presentation and analysis of results

- Static
- Dynamic
  - Implicit
  - Explicit
- Visco
- Heat transfer
  - Steady state
  - Transient
- Coupled temperature-displacement
- Buckling
- Electromagnetism
- Fluid Flow

- Linear
- Nonlinear

FEM simulation steps

1. Geometry
   - geometrical simplifications
   - reduction of dimensions
2. Physical properties
   - material properties
   - structural properties
3. Geometrical discretization
   - element type
   - meshing
4. Type of analysis
5. **Loading and Boundary/Initial conditions**
6. Presentation and analysis of results

[1]

[1] Emri, I., Voloshin, A., 2016, *Statics – Learning from Engineering Examples, Springer Science*, doi: 10.1007/978-1-4939-2101-0

FEM simulation steps

1. Geometry
   - geometrical simplifications
   - reduction of dimensions
2. Physical properties
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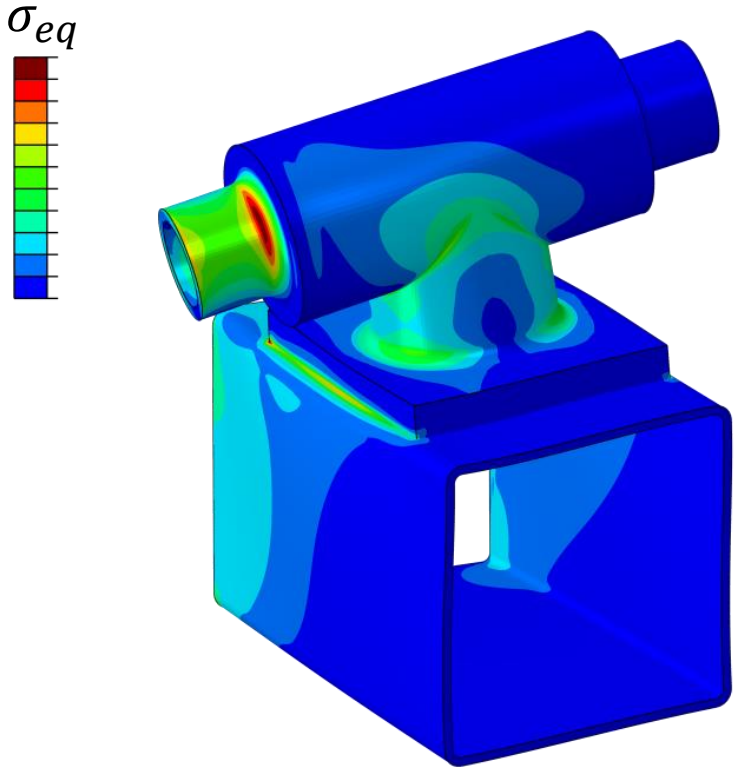
6. **Presentation and analysis of results**

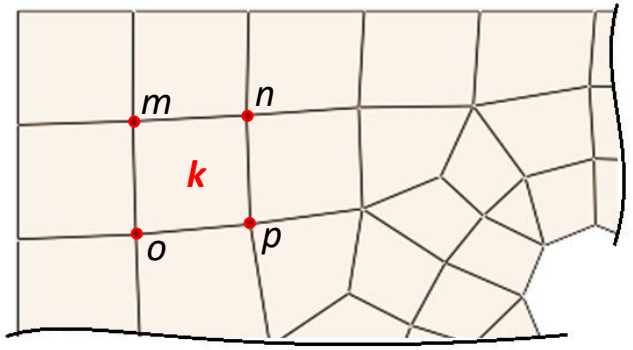


Defining a simulation

- Geometry
- Sets
- Material behaviour
- Type of analysis
- Solver type
- Loading, Boundary/Initial conditions
- Output

Defining a simulation

- **Geometry**
- Sets
- Material behaviour
- Type of analysis
- Solver type
- Loading, Boundary/Initial conditions
- Output

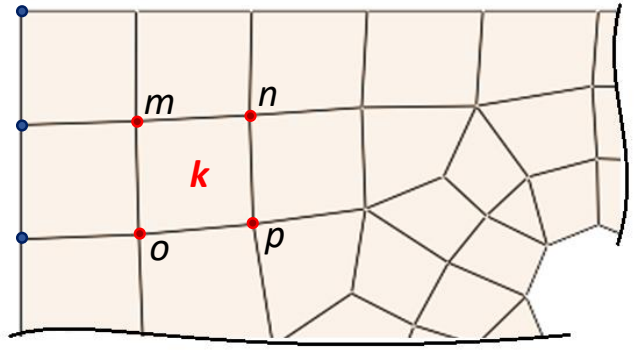



```
*Node
      1,          10.,           5.
      2,         -25.,           5.
      3,         -25.,         -15.
      4,          15.,         -15.
      5,          15.,           0.
      6,          15.,          10.
      7,          15.,          25.
      8,         -25.,          25.
      9,          55.,           5.
     10,          20.,           5.
```

```
*Element, type=CPS8
   1,  675,  708,  644, 1235
   2,  602,  131,  132,  589
   3,  123,  646,  672,  122
   4,  116,  608,  607,  115
   5,  595,  108,  109,  643
   6,  112,  584,  597,  111
   7,  575,   78,   79,  576
   8,  536,  574,  575,  537
```

Defining a simulation

- Geometry
- **Sets**
- Material behaviour
- Type of analysis
- Solver type
- Loading, Boundary/Initial conditions
- Output




```
*Nset, nset=Load
    2,    3,    8,   57,   58,   59,   60,   61,
  221,  222,  223,  224,  225,  226, 4091, 4094,
```

```
*Elset, elset=steel
   20,   21,   22,   82,   90,  100,  101,  102,
 1148, 1149, 1150, 1154, 1155
```

Defining a simulation

- Geometry
- Sets
- **Material behaviour**
- Type of analysis
- Solver type
- Loading, Boundary/Initial conditions
- Output

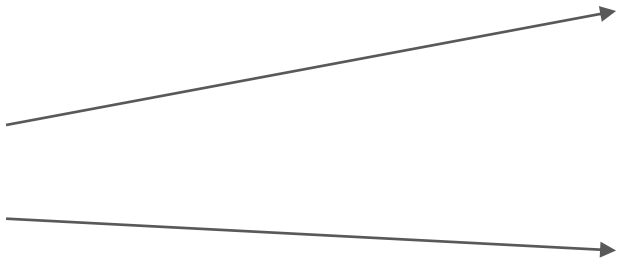
```
*Material, name=Steel
*Elastic
 2.1e+08, 0.3
*Plastic
 284000.,    0.
 300000., 0.01
 310000., 0.02
 350000., 0.03
 400000., 0.05
```

```
*Material, name=Membrane
*Expansion
 0.00016,
*Hyperelastic, moduli=LONG TER
 7e+06, 5e+06, 1e-09
*Viscoelastic, time=PRONY
 0.9,    0.,6300.
```

```
      SUBROUTINE UHYPER(BI1,BI2,AJ,U,UI1,UI2,UI3,TEMP,NOEL,
     1 CMNAME,INCMPFLAG,NUMSTATEV,STATEV,NUMFIELDV,FIELDV,
     2 FIELDVINC,NUMPROPS,PROPS)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CMNAME
      DIMENSION U(2),UI1(3),UI2(6),UI3(6),STATEV(*),FIELDV(*),
     2 FIELDVINC(*),PROPS(*)
      real*8 ac, bc, E
c
      ac=props(1)
      bc=props(2)
      E=2.71828182845905
c
      U(1)=(ac*(-1. + E**((bc*(-3. + BI1))/2.)))/bc
c
      UI1(1)=(ac*E**((bc*(-3. + BI1))/2.))/2.
      UI1(2)=0.
      UI1(3)=0.
c
      UI2(1)=(ac*bc*E**((bc*(-3. + BI1))/2.))/4.
      UI2(2)=0.
      UI2(3)=0.
      UI2(4)=0.
      UI2(5)=0.
      UI2(6)=0.
c
      UI3(1)=0.
      UI3(2)=0.
      UI3(3)=0.
      UI3(4)=0.
      UI3(5)=0.
      UI3(6)=0.
c
      RETURN
      END
```

Defining a simulation

- Geometry
- Sets
- Material behaviour
- **Type of analysis**
- **Solver type**
- Loading, Boundary/Initial conditions
- Output



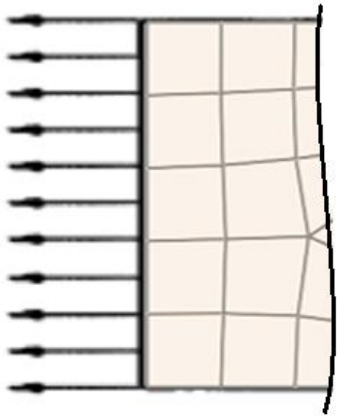
Static, Dynamic (Implicit, Explicit),
Visco, Thermal, Coupled thermal-displacement,
Linear/Nonlinear
…

Equation solver
Solution Techniques
Incrementation
Convergence tolerances
…

Defining a simulation

- Geometry
- Sets
- Material behaviour
- Type of analysis
- Solver type
- **Loading, Boundary/Initial conditions**
- Output

```
*Boundary, op=NEW
edge_L, 3, 3
*Boundary, op=NEW
edge_R, 3, 3
*Dsload, op=NEW
inner, P, 13.9988
```



Defining a simulation

- Geometry
- Sets
- Material behaviour
- Type of analysis
- Solver type
- Loading, Boundary/Initial conditions
- **Output**

```
*Output, field
*Node Output
CF, COORD, RF, U
*Element Output, directions=YES
LE, P, PE, PEEQ, PEMAG, S, TEMP
*Output, history
*Node Output, nset=u6
COOR1, COOR2, COOR3
```

Thank you for your attention!

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